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SPIN MICROSCOPY WITH ENHANCED WILSON LINES IN THE TMD PARTON DENSITIES[1]

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Abstract

We discuss the possibility of non-minimal gauge invariance of transverse-momentum-dependent parton densities (TMDs) that allows direct access to the spin degrees of freedom of fermion fields entering the operator definition of (quark) TMDs. This is achieved via enhanced Wilson lines that are supplied with the spin-dependent Pauli term $\sim F^{\mu\nu}[\gamma_\mu, \gamma_\nu]$, thus providing an appropriate tool for the "microscopic" investigation of the spin and color structure of TMDs. We show that this generalization leaves the leading-twist TMD properties unchanged but modifies those of twist three by contributing to their anomalous dimensions. We also comment on Collins' recent criticism of our approach.

Precise knowledge of the geometrical structure, as well as of the spin and color properties, of the Wilson lines (gauge links) in the operator formulation of TMDs is an essential ingredient of the QCD factorization approach to semi-inclusive hadronic processes [1, 2]. The path-[$\mathcal{C}$]-dependent non-Abelian gauge links $[y; x|\mathcal{C}] \equiv \mathcal{P} \exp\left[-ig \int_{x[\mathcal{C}]}^{y} dz^\mu A_\mu^a(z) t^a\right]$, which ensure the gauge invariance of nonlocal operator products and correlators, are intimately related to important issues of TMDs, like the ultraviolet (UV) and rapidity evolution equations, the generation of $T-$odd effects, the proof or violation of factorization, etc. [2, 3]. Different operator definitions of the TMDs can comprise bunches of longitudinal and transverse gauge links possessing a quite involved space-time structure, with non-trivial properties in color space as well (see, e.g., [3–8] and further discussions and references cited therein). Moreover, the location of the gauge integration contours in the $(z^+, z^-, \boldsymbol{z}_\perp)$-plane (in contrast to collinear PDFs, where they belong to a single lightlike ray and are, therefore, one-dimensional) necessitates the inclusion of (possible) contributions of *non-minimal* spin-dependent terms, expressed in terms of enhanced Wilson lines (more below). The path-dependence, being in some sense "hidden" in the case of collinear PDFs [7], becomes a key issue in TMDs. In particular, explicit spin-dependent terms in the gauge links can create significant effects in lattice simulations [2, 9], depending on the geometry of the integration paths, and may also affect the TMD-factorization properties [3].

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To this end, we introduced in [10] an enhanced gauge link, denoted by [[...]], which contains the Pauli term proportional to the gluon strength tensor $\sim F^a_{\mu\nu} J_{\mu\nu} = (1/4) F^a_{\mu\nu}[\gamma_\mu, \gamma_\nu]$. This is the simplest example to realize a direct product of two orthogonal "spaces": The first "space" is the color one, with the *minimal* Wilson lines in the fundamental or adjoint representation of $SU(3)_c$. In the second "space", the spin correlations are generated by the Pauli terms [10]. The spin-dependent terms yield next-to-leading-order twist effects with respect to the spin-"blind" ones, as it follows from usual power-counting.

We discuss below, the main results of our study of the renormalization-group properties of the TMD distribution functions with enhanced gauge-link insertions [10], focusing on the UV properties of the "quark-in-a-quark" TMD. According to our generalized concept of gauge invariance, the *unsubtracted* distribution function of a quark with momentum k and flavor a in a quark with momentum p reads

$$\mathcal{F}^\Gamma_a(x, \boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\int dk^- \int \frac{d^4\xi}{(2\pi)^4}\, \mathrm{e}^{-ik\cdot\xi} \langle p, s\ | \bar{\psi}_a(\xi) [[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]]^\dagger$$
$$\times [[\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]]^\dagger \Gamma\ [[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]][[\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]]\psi_a(0)|p, s\rangle\ , \quad (1)$$

where Γ stands for the Dirac structure constructed from one or several γ-matrices. The matrix elements interpolate between the one-fermion states with momentum p and spin s: $|p, s\rangle$. In the tree-approximation one has

$$\mathcal{F}^{\Gamma(0)}(x, \boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[(\hat{p} + m)\,(1 + \gamma_5\hat{s})\ \Gamma\right]\ \delta(p^+ - xp^+)\delta^{(2)}(\boldsymbol{k}_\perp)\ . \quad (2)$$

For the unpolarized TMD PDF with $\Gamma = \gamma^+$, one obtains the (twist-two) result

$$\mathcal{F}^{\gamma^+(0)}(x, \boldsymbol{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[(\hat{p} + m)\,(1 + \gamma_5\hat{s})\,\gamma^+\right]\delta(p^+ - xp^+)\delta^{(2)}(\boldsymbol{k}_\perp) = \delta(1 - x)\delta^{(2)}(\boldsymbol{k}_\perp)\ . \quad (3)$$

The helicity and the transversity distributions read, respectively,

$$\mathcal{F}^{\gamma^+\gamma_5(0)}(x, \boldsymbol{k}_\perp) = \delta(1 - x)\delta^{(2)}(\boldsymbol{k}_\perp)\cdot\lambda\ ,\ \mathcal{F}^{i\sigma^{i+}\gamma_5(0)}(x, \boldsymbol{k}_\perp) = \delta(1 - x)\delta^{(2)}(\boldsymbol{k}_\perp)\cdot\boldsymbol{s}^{\boldsymbol{i}}_\perp\ , \quad (4)$$

where λ is the helicity and $\boldsymbol{s}^{\boldsymbol{i}}_\perp$ is the transverse spin of parton i. Note that the above normalization conditions can only be obtained within the quantization procedure in the light-cone gauge, where the (minimal) longitudinal Wilson lines vanish and the equal-time canonical commutation relations for the quark creation and annihilation operators $\{a^\dagger(k, \lambda), a(k, \lambda)\}$ are consistent with the *parton-number interpretation* of the TMD in the tree-approximation (see [11] for more): $\mathcal{F}^{(0)}(x, \boldsymbol{k}_\perp) \sim \langle p|a^\dagger(k^+, \boldsymbol{k}_\perp; \lambda)\ a(k^+, \boldsymbol{k}_\perp; \lambda)|p\rangle$. In line with the above explanations, we define a generic enhanced gauge link evaluated along some fixed but else arbitrary direction w from zero to infinity according to

$$[[\infty; 0]] = \mathcal{P}\exp\left[-ig\int_0^\infty d\sigma\ w_\mu\ A^\mu_a(w\sigma)t^a - ig\int_0^\infty d\sigma\ J_{\mu\nu}F^{\mu\nu}_a(w\sigma)t^a\right]\ , \quad (5)$$

where the four-vector w may be longitudinal (light-like) $w_L = n^-$, or transverse $w_T = (0^+, 0^-, \boldsymbol{l}_\perp)$. The enhanced Wilson lines (5) significantly enlarge the gauge-invariant formalism of quark and gluon operators entering the TMD correlators.

To investigate the structure of the UV singularities in the leading α_s-order, we evaluate all graphs contributing to this order given in [10], where one can also find the technical

details and the appropriate Feynman rules. Note that there are two different perturbative expansions in the generalized TMD given by (1): one stems from the Heisenberg quark field operators, i.e., $\psi_a(\xi) = \mathrm{e}^{-ig\left[\int d\eta\ \bar{\psi}\hat{\mathcal{A}}\psi\right]}\ \psi_a^{\mathrm{free}}(\xi)$, $\int dx\ \bar{\psi}\hat{\mathcal{A}}\psi\ \equiv \int d^4x\ \bar{\psi}(x)\gamma_\mu\psi(x)\mathcal{A}^\mu(x)$. The other originates from the evaluation of the product of the enhanced gauge links up to $\mathcal{O}(g^2)$. Applying the light-cone gauge $A^+ = 0$) one has

$$[[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \mathbf{0}_\perp]] \cdot [[\infty^-, \mathbf{0}_\perp; 0^-, \mathbf{0}_\perp]] = 1 - ig\left(\mathcal{U}_1 + \mathcal{U}_2 + \mathcal{U}_3\right) - g^2\left(\mathcal{U}_4 + \mathcal{U}_5 + \ldots \mathcal{U}_{10}\right)\ , \tag{6}$$

where the individual contributions $\mathcal{U}_i$ have to be contracted with themselves as well as with corresponding terms in the Heisenberg field operators.

The singularity structure of the twist-two TMD with the Dirac structures $\Gamma_{\mathrm{tw}-2} = \{\gamma^+, \gamma^+\gamma^5, i\sigma^{i+}\gamma^5\}$, cancel by the Hermitean conjugated (mirror) diagrams, in contrast to the twist-three TMDs (e.g., $\Gamma_{\mathrm{tw}-3} = \gamma^i$) which receive non-trivial UV divergent contributions from the Pauli term, like

$$\Gamma_{\mathrm{tw}-3}\langle \mathbf{A}^\perp F^-\rangle + \langle \mathbf{A}^\perp F^-\rangle\Gamma_{\mathrm{tw}-3} = -C_{\mathrm{F}}\ \frac{1}{4\pi}[\gamma^+, \gamma^-]\ \Gamma(\varepsilon)\ \left(4\pi\frac{\mu^2}{\lambda^2}\right)^{\varepsilon}\ . \tag{7}$$

Here, $\langle \mathbf{A}^\perp F^-\rangle$ denotes the result stemming from the cross-talk between the minimal transverse gauge link and the enhanced longitudinal gauge link containing a Pauli term. In order to render the TMD singularity-free, one has to handle the overlapping UV and rapidity divergences induced by the gluon propagator in the lightcone gauge. To this end, we refurbished in [7] the untruncated definition in Eq. 1 by a soft renormalization factor along a particular gauge contour going off the lightcone. This soft factor takes care of the overlapping UV and infrared (rapidity) divergences which cannot be regularized dimensionally, as in the case of purely longitudinal gauge links—see [12] and references cited therein.

Recently, Collins [4] questioned the validity of this definition and proposed another one. He argues that the gluon propagator in the lightcone gauge subject to the Mandelstam-Leibbrandt (ML) boundary prescription, $D_{\mathrm{ML}}^{\mu\nu}$, is not transverse, i.e., $n_\mu D_{\mathrm{ML}}^{\mu\nu} \neq 0$. The propagator displayed by Collins as Eq. (15) in [4] is *not* the ML one but the result of using the Principal-Value prescription. This propagator, as well as the Retarded and the Advanced one, are indeed not transverse. In contrast, the *correct* ML propagator (see last entry in [7]) *is* transverse and the soft factor reduces to unity. The second argument by Collins is that the graphs shown in Eq. (16) in [4] give uncanceled rapidity divergences. If the displayed graphs are to be evaluated in the lightcone gauge, as used in our works in [7] and in [10], then they both vanish. In a general covariant gauge, these graphs contribute singularities that are indispensable in order to cancel those singular terms, induced by the gluon propagator, which contain the gauge parameter. There are no surviving singularities.

In conclusion, we discussed a new operator formulation of gauge-invariant TMDs which provides direct access to the spin degrees of freedom of the partonic fields by means of the Pauli term in the gauge links, hence allowing a microscopic analysis of the spin-color structure of TMDs relevant for phenomenology.

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